All books and notes must be closed. No mobile phones should be on your person or even close to you. Leave your bags and phones by the teacher's desk.

UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

Final exam, 22 Jan. 2018 – Duration 2 hours.

All curves are positively oriented unless otherwise noted.

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1. (2 units) Prove that every bounded entire function f is a constant. *Hint*: Assume that there are two different points $a, b \in \mathbb{C}$ with $f(a) \neq f(b)$. Take a very big circle $|z| = R$ and apply Cauchy's formula for the values of f at a, b.

2. (1 unit) Compute the integral

$$\oint_C \frac{z^3 + z^2}{(z-w)^3}\, dz$$

where C is a simple closed curve with point w in its interior.

3. (1 unit) Prove that if f is entire and we have $|f(x+iy)| \le e^x$ for all $x, y \in \mathbb{R}$ then $f(z) = Ce^z$ for some constant C with $|C| \le 1$.

4. (1 unit) Find all Laurent expansions of $f(z) = (2-z)^{-1}$ centered at 0. Explain in which region each of them is valid. Do not bother with the boundary.

5. (1 unit) The function e^z is entire and has the property $(e^z)' = e^z$. Using only this derive the power series of e^z centered at 0.

6. (1 unit) After you find the constants A and B such that

$$\frac{1}{z^2+1} = \frac{A}{z+i} + \frac{B}{z-i}$$

compute $\displaystyle\oint_{|z|=2} \frac{ze^z}{z^2+1} dz$.

7. (1 unit) If the f_n are entire functions and they converge to f uniformly in the region $\{z : \ |z| \le 10\}$ show that f is analytic in the region $\{z : \ |z| < 1\}$.

8. (2 units) Compute the integral

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^4}$$

using residue theory. Explain all your steps.

Complex Analysis - Fall 2018-19 - M. Kolountzakis