
Turn in your solutions in class on Thursday 20/2/2020. Write briefly without ommitting the essentials.

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1. Two functions $\mathbb{R} \to \mathbb{C}$ are considered "identical" if they differ on a set of measure 0. Show that this is an equivalence relation. Show next that if $1 \le p < \infty$ and $f, g \in L^p(\mathbb{R})$ with $\|f-g\|_p = 0$ then f, g are "identical".

2. If $m(A) = 1$ and $f \in L^\infty(A)$ show that $\lim_{p\to\infty} \|f\|_p = \|f\|_\infty$.

💡Let $\epsilon > 0$ and
$$E = \{x \in A : |f(x)| \ge (1-\epsilon)\|f\|_\infty\}.$$
Then $m(E) > 0$ (otherwise esssup$|f|$ would be smaller) and $\|f\|_p \ge (\int_E |f|^p)^{1/p}$.