

Turn in your solutions in class on Thursday 27/2/2020. Write briefly without omitting the essentials.

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1. (i) If  $a_n, b_n, n = 1, 2, \dots, N$  are complex numbers and  $B_k = \sum_{n=1}^k b_n$  show the very useful formula of summation by parts (which is the analogue of intergration by parts for sums)

$$\sum_{n=M}^N a_n b_n = a_N B_N - a_M B_{M-1} - \sum_{n=M}^{N-1} (a_{n+1} - a_n) B_n.$$

- (ii) If  $a_n \rightarrow 0$  is a decreasing sequence and the partial sums of the series  $\sum_n b_n$  are bounded then the series  $\sum_n a_n b_n$  converges.

2. If  $0 < \delta < \pi$  compute the Fourier coefficients of the function  $f : [-\pi, \pi] \rightarrow \mathbb{R}$  given by the formula

$$f(x) = \begin{cases} 1 - \frac{|x|}{\delta} & (|x| \leq \delta) \\ 0 & (\delta \leq |x| \leq \pi). \end{cases}$$