

1. Prove that  $e^x = 1 + x + x^2/2 + O(x^3)$  for  $x \rightarrow 0$ .

2. If  $a_n = O(n)$  show that  $\sum_{k=1}^n a_k = O(n^2)$ .

3. If  $f(x) = o(x)$  for  $x \rightarrow 0$  show that

$$\frac{1}{1 - f(x)} = 1 + o(x).$$

4. If  $\epsilon > 0$  show that  $\log x = o(x^\epsilon)$  for  $x \rightarrow \infty$ .

5. Show that  $\sqrt{x + \sqrt{x}} = (1 + o(1))x^{1/4}$  for  $x \rightarrow 0^+$ .

6. Exactly one of the following relations is correct. Which one and why?

$$(a) \ 2^{o(n)} = o(2^n), \quad (b) \ 2^{O(n)} = O(2^n).$$