Η μέση τιμή και διασπορά της γεωμετρικής κατανομής

$X \sim \text{Γεωμ}(p) \quad p \in (0,1)$

Νόμισμα συνεχεια
πιθ. ΚΟΡ = p
X = ο χρόνος της
1ης κορώνας

$$f_X(k) = \mathbb{P}(X = k) = \begin{cases} 0 & k \leq 0 \\ p(1-p)^{k-1} & k \geq 1 \end{cases}$$

$$\mathbb{E}X = 1/p$$

$$\sigma^2(X) = \frac{1-p}{p^2}$$

$$\mathbb{E}X = \sum_{k=1}^{\infty} k\,p\,(\underbrace{1-p}_{q})^{k-1} = \sum_{k=1}^{\infty} k\,p\,q^{k-1} =$$

$$= p\sum_{1}^{\infty} k q^{k-1} = p \sum_{1}^{\infty} (q^k)'$$ → παράγωγος ως προς q

$$= p\left(1+\sum_{1}^{\infty} q^k\right)' = p\left(\frac{1}{1-q}\right)' = p\frac{1}{(1-q)^2}$$

$$\sum_{0}^{\infty} x^k = \frac{1}{1-x} \quad |x|<1$$

$$= \frac{p}{p^2} = 1/p$$

$$\sigma^2(X) = \mathbb{E}(X^2) - (\mathbb{E}X)^2 = \frac{2-p}{p^2} - \frac{1}{p^2} = \frac{1-p}{p^2}$$

$$\mathbb{E}X^2 = \sum_{1}^{\infty} k^2 p q^{k-1} = p \sum_{1}^{\infty} k^2 q^{k-1} = p\,\frac{2-p}{p^3} = \frac{2-p}{p^2}$$

$p+q=1$

$$\frac{1}{1-q} = \sum_{0}^{\infty} q^k \Rightarrow \frac{1}{(1-q)^2} = \sum_{1}^{\infty} k q^{k-1} \Rightarrow \frac{q}{(1-q)^2} = \sum_{1}^{\infty} k q^k$$

$$\xRightarrow{\frac{d}{dq}} \frac{(1-q)^2 + 2q(1-q)}{(1-q)^4} = \sum_{1}^{\infty} k^2 q^{k-1} \Rightarrow \frac{1+q}{(1-q)^3} = \frac{2-p}{p^3} = \sum k^2 p^{k-1}$$