Μέση τιμή και διασπορά της κατανομής Poisson(λ) $\lambda > 0$

$$f_X(k) = \begin{cases} 0 , & k<0 \\ e^{-\lambda}\dfrac{\lambda^k}{k!} , & k \geq 0 \end{cases}$$

$$\sum_{k\in\mathbb{Z}} f_X(k) = \sum_{k=0}^{\infty} e^{-\lambda}\frac{\lambda^k}{k!} = e^{-\lambda}\underbrace{\sum_{k=0}^{\infty}\frac{\lambda^k}{k!}}_{=e^{\lambda}} = 1 \qquad \forall \lambda \in \mathbb{R}$$

$$\overset{\lambda}{\mathbb{E}X}, \quad \overset{\lambda}{\sigma^2(X)}$$

$$\mathbb{E}X = \sum_{k=0}^{\infty} k f_X(k) = \sum_{k=0}^{\infty} k e^{-\lambda}\frac{\lambda^k}{k!} = e^{-\lambda}\sum_{k=1}^{\infty} k\frac{\lambda^k}{k!} = e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^k}{(k-1)!} =$$

$$\underset{v=k-1}{=} e^{-\lambda}\sum_{v=0}^{\infty}\frac{\lambda^{v+1}}{v!} = e^{-\lambda}\lambda\sum_{v=0}^{\infty}\frac{\lambda^v}{v!} = e^{-\lambda}\lambda e^{\lambda} = \lambda$$

$$\sigma^2(X) = \mathrm{Var}X = \mathbb{E}(X^2) - (\mathbb{E}X)^2 = \overset{\lambda^2+\lambda}{\mathbb{E}(X^2)} - \lambda^2 = \lambda$$

$$\mathbb{E}(X^2) = e^{-\lambda}\sum_{k=1}^{\infty} k^2\frac{\lambda^k}{k!} = e^{-\lambda}\sum_{k=1}^{\infty}\overset{k-1+1}{k}\frac{\lambda^k}{(k-1)!} = e^{-\lambda}\sum_{k=2}^{\infty}(k-1)\frac{\lambda^k}{(k-1)!} + \overbrace{e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^k}{(k-1)!}}^{\lambda}$$

$$= e^{-\lambda}\sum_{k=2}^{\infty}\frac{\lambda^k}{(k-2)!} + \lambda \underset{t=k-2}{=} e^{-\lambda}\lambda^2\sum_{t=0}^{\infty}\frac{\lambda^t}{t!} + \lambda = \lambda^2+\lambda$$